H 1423 Q4 Ctd.

(iii) $\frac{KE(A)}{KE(B)} = \frac{7}{1}$ $\Rightarrow$ $\frac{\frac{1}{2}ma^2 + \frac{1}{2}mb^2}{\frac{1}{2}mX^2} = \frac{7}{1}$

①, ④, ⑤ $\Rightarrow$ $\frac{\left[\frac{u}{4}(1-e)\right]^2 + \left[\frac{\sqrt{3}}{2}u\right]^2}{\left[\frac{u}{4}[1+e)\right]^2} = \frac{7}{1}$

$\therefore \frac{1}{16}(1-e)^2 + \frac{3}{4} = \frac{7}{16}(1+e)^2$

$\Rightarrow (1-e)^2 + 12 = 7(1+e)^2$

$\therefore 1 - 2e + e^2 + 12 = 7 + 14e + 7e^2$

$\Rightarrow 6e^2 + 16e - 6 = 0$

$\Rightarrow 3e^2 + 8e - 3 = 0$

$\Rightarrow (3e-1)(e+3) = 0$

$\Rightarrow 3e - 1 = 0$ or $e + 3 = 0$

$\Rightarrow e = \frac{1}{3}$ ✓ or $e = -3$ ← impossible physically

Footnotes:

* $\vec{u} = u\cos 60\,\vec{i} + u\sin 60$

$|\vec{u}|^2 = (u\cos 60)^2 + (u\sin 60)^2$

$= u^2(\cos^2 60 + \sin^2 60)$

$= u^2(1)$

$= u^2$

So $\frac{1}{2}m(u\cos 60)^2 + \frac{1}{2}m(u\sin 60)^2 = \frac{1}{2}mu^2$.

Or to put it another way:

Writing $\vec{u} = u\cos 60\,\vec{i} + u\sin 60\,\vec{j}$ tells us magnitude of vector is u.

and strictly speaking

KE (A) before collision $= \frac{1}{2}m|\vec{u}|^2$

$= \frac{1}{2}mu^2$.

** Note I wrote $\vec{v}_1$ like this as I find it easier to cope with than $\vec{v}_1 = v_1\cos\theta\,\vec{i} + v_2\sin\theta\,\vec{j}$.

N.B. Key was to calculate a, b and x ie $\vec{v}_1$ and $\vec{v}_2$ as usual!